In[1]:=
```
(*For faster calculation, assume n=5 scintillators (in the experiment there were 10)*)
n = 5;
(*R1 = Scintillator response (T1, 1 photon) / X,
where X = M (measured data) or δM (measurement error)*)
R1 = Array[r1, n]
(*The same for T2*)
R2 = Array[r2, n]
(*The same for T3*)
R3 = Array[r3, n]
(*Q=M/X; NB: Q = {1,1,1,1,1} for X = M*)
Q = Array[q, n]
```

Out[2]= {r1[1], r1[2], r1[3], r1[4], r1[5]}

Out[3]= {r2[1], r2[2], r2[3], r2[4], r2[5]}

Out[4]= {r3[1], r3[2], r3[3], r3[4], r3[5]}

Out[5]= {q[1], q[2], q[3], q[4], q[5]}

In[6]:=
```
(*For three temperatures*)
```

In[7]:=
```
dif = Q - n1 R1 - n2 R2 - n3 R3
mf3 = dif.dif
```

Out[7]= {q[1] - n1 r1[1] - n2 r2[1] - n3 r3[1],
q[2] - n1 r1[2] - n2 r2[2] - n3 r3[2], q[3] - n1 r1[3] - n2 r2[3] - n3 r3[3],
q[4] - n1 r1[4] - n2 r2[4] - n3 r3[4], q[5] - n1 r1[5] - n2 r2[5] - n3 r3[5]}

Out[8]= (q[1] - n1 r1[1] - n2 r2[1] - n3 r3[1])^2 +
(q[2] - n1 r1[2] - n2 r2[2] - n3 r3[2])^2 + (q[3] - n1 r1[3] - n2 r2[3] - n3 r3[3])^2 +
(q[4] - n1 r1[4] - n2 r2[4] - n3 r3[4])^2 + (q[5] - n1 r1[5] - n2 r2[5] - n3 r3[5])^2

In[9]:=
```
d1 = 1/2 Simplify[∂n1 mf3]
d2 = 1/2 Simplify[∂n2 mf3]
d3 = 1/2 Simplify[∂n3 mf3]
```

Out[9]=
$$-q[1]\times r1[1]-q[2]\times r1[2]-q[3]\times r1[3]-q[4]\times r1[4]-q[5]\times r1[5]+ n1\left(r1[1]^2+r1[2]^2+r1[3]^2+r1[4]^2+r1[5]^2\right)+n2\,r1[1]\times r2[1]+n2\,r1[2]\times r2[2]+ n2\,r1[3]\times r2[3]+n2\,r1[4]\times r2[4]+n2\,r1[5]\times r2[5]+n3\,r1[1]\times r3[1]+ n3\,r1[2]\times r3[2]+n3\,r1[3]\times r3[3]+n3\,r1[4]\times r3[4]+n3\,r1[5]\times r3[5]$$

Out[10]=
$$-q[1]\times r2[1]+n2\,r2[1]^2-q[2]\times r2[2]+n2\,r2[2]^2- q[3]\times r2[3]+n2\,r2[3]^2-q[4]\times r2[4]+n2\,r2[4]^2-q[5]\times r2[5]+n2\,r2[5]^2+ n1\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])+ n3\,r2[1]\times r3[1]+n3\,r2[2]\times r3[2]+n3\,r2[3]\times r3[3]+n3\,r2[4]\times r3[4]+n3\,r2[5]\times r3[5]$$

Out[11]=
$$-q[1]\times r3[1]+n2\,r2[1]\times r3[1]+n3\,r3[1]^2-q[2]\times r3[2]+n2\,r2[2]\times r3[2]+ n3\,r3[2]^2-q[3]\times r3[3]+n2\,r2[3]\times r3[3]+n3\,r3[3]^2-q[4]\times r3[4]+ n2\,r2[4]\times r3[4]+n3\,r3[4]^2-q[5]\times r3[5]+n2\,r2[5]\times r3[5]+n3\,r3[5]^2+ n1\,(r1[1]\times r3[1]+r1[2]\times r3[2]+r1[3]\times r3[3]+r1[4]\times r3[4]+r1[5]\times r3[5])$$

In[12]:=
```
Collect[d1, {n1, n2, n3}]
Print[]
Collect[d2, {n1, n2, n3}]
Print[]
Collect[d3, {n1, n2, n3}]
```

Out[12]=
$$-q[1]\times r1[1]-q[2]\times r1[2]-q[3]\times r1[3]-q[4]\times r1[4]- q[5]\times r1[5]+n1\left(r1[1]^2+r1[2]^2+r1[3]^2+r1[4]^2+r1[5]^2\right)+ n2\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])+ n3\,(r1[1]\times r3[1]+r1[2]\times r3[2]+r1[3]\times r3[3]+r1[4]\times r3[4]+r1[5]\times r3[5])$$

Out[14]=
$$-q[1]\times r2[1]-q[2]\times r2[2]-q[3]\times r2[3]-q[4]\times r2[4]-q[5]\times r2[5]+ n1\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])+ n2\left(r2[1]^2+r2[2]^2+r2[3]^2+r2[4]^2+r2[5]^2\right)+ n3\,(r2[1]\times r3[1]+r2[2]\times r3[2]+r2[3]\times r3[3]+r2[4]\times r3[4]+r2[5]\times r3[5])$$

Out[16]=
$$-q[1]\times r3[1]-q[2]\times r3[2]-q[3]\times r3[3]-q[4]\times r3[4]-q[5]\times r3[5]+ n1\,(r1[1]\times r3[1]+r1[2]\times r3[2]+r1[3]\times r3[3]+r1[4]\times r3[4]+r1[5]\times r3[5])+ n2\,(r2[1]\times r3[1]+r2[2]\times r3[2]+r2[3]\times r3[3]+r2[4]\times r3[4]+r2[5]\times r3[5])+ n3\left(r3[1]^2+r3[2]^2+r3[3]^2+r3[4]^2+r3[5]^2\right)$$

In[17]:=
```
(*Check*)
Simplify[d1 == -Q.R1 + n1 R1.R1 + n2 R1.R2 + n3 R1.R3]
Simplify[d2 == -Q.R2 + n1 R2.R1 + n2 R2.R2 + n3 R2.R3]
Simplify[d3 == -Q.R3 + n1 R3.R1 + n2 R3.R2 + n3 R3.R3]
```

Out[17]= True

Out[18]= True

Out[19]= True

In[20]:=
```
s3 = FullSimplify[Solve[{
      QR1 == n1 R1R1 + n2 R1R2 + n3 R1R3,
      QR2 == n1 R1R2 + n2 R2R2 + n3 R2R3,
      QR3 == n1 R1R3 + n2 R2R3 + n3 R3R3},
     {n1, n2, n3}]][[1]]
```

Out[20]=
$$\left\{n1 \to \frac{QR3\ R1R3\ R2R2 - QR3\ R1R2\ R2R3 - QR2\ R1R3\ R2R3 + QR1\ R2R3^2 + QR2\ R1R2\ R3R3 - QR1\ R2R2\ R3R3}{R1R3^2\ R2R2 - 2\ R1R2\ R1R3\ R2R3 + R1R1\ R2R3^2 + R1R2^2\ R3R3 - R1R1\ R2R2\ R3R3},\right.$$
$$n2 \to \frac{-QR3\ R1R2\ R1R3 + QR2\ R1R3^2 + QR3\ R1R1\ R2R3 - QR1\ R1R3\ R2R3 - QR2\ R1R1\ R3R3 + QR1\ R1R2\ R3R3}{R1R3^2\ R2R2 - 2\ R1R2\ R1R3\ R2R3 + R1R2^2\ R3R3 + R1R1\ \left(R2R3^2 - R2R2\ R3R3\right)},$$
$$\left. n3 \to \frac{QR3\ R1R2^2 - QR2\ R1R2\ R1R3 - QR3\ R1R1\ R2R2 + QR1\ R1R3\ R2R2 + QR2\ R1R1\ R2R3 - QR1\ R1R2\ R2R3}{R1R3^2\ R2R2 - 2\ R1R2\ R1R3\ R2R3 + R1R1\ R2R3^2 + R1R2^2\ R3R3 - R1R1\ R2R2\ R3R3}\right\}$$

In[21]:=
```
(*Check the solution with the substitututions*)
subst = {QR1 → Q.R1, QR2 → Q.R2, QR3 → Q.R3, R1R1 → R1.R1,
   R1R2 → R1.R2, R1R3 → R1.R3, R2R2 → R2.R2, R2R3 → R2.R3, R3R3 → R3.R3};
Simplify[d1 /. s3 /. subst]
Simplify[d2 /. s3 /. subst]
Simplify[d3 /. s3 /. subst]
```

Out[22]= 0

Out[23]= 0

Out[24]= 0

In[25]:=
```
(*For c/c++ codes*)
CForm[n1 /. s3]
```

Out[25]//CForm=
```
(QR3*R1R3*R2R2 - QR3*R1R2*R2R3 - QR2*R1R3*R2R3 + QR1*Power(R2R3,2) + QR2*R1R2*R3R3 -
     QR1*R2R2*R3R3)/
   (Power(R1R3,2)*R2R2 - 2*R1R2*R1R3*R2R3 + R1R1*Power(R2R3,2) + Power(R1R2,2)*R3R3 -
     R1R1*R2R2*R3R3)
```

In[26]:=
```
CForm[n2 /. s3]
```

Out[26]//CForm=
```
(-(QR3*R1R2*R1R3) + QR2*Power(R1R3,2) + QR3*R1R1*R2R3 - QR1*R1R3*R2R3 - QR2*R1R1*R3R3 +
     QR1*R1R2*R3R3)/
   (Power(R1R3,2)*R2R2 - 2*R1R2*R1R3*R2R3 + Power(R1R2,2)*R3R3 +
     R1R1*(Power(R2R3,2) - R2R2*R3R3))
```

In[27]:=
```
CForm[n3 /. s3]
```

Out[27]//CForm=
```
(QR3*Power(R1R2,2) - QR2*R1R2*R1R3 - QR3*R1R1*R2R2 + QR1*R1R3*R2R2 + QR2*R1R1*R2R3 -
     QR1*R1R2*R2R3)/
   (Power(R1R3,2)*R2R2 - 2*R1R2*R1R3*R2R3 + R1R1*Power(R2R3,2) + Power(R1R2,2)*R3R3 -
     R1R1*R2R2*R3R3)
```

In[28]:=
```
(*Check that the denominators in the equations for n1, n2, n3 are equal*)
Simplify[Denominator[n1 /. s3] == Denominator[n2 /. s3]]
```

Out[28]= True

In[29]:=
```
Simplify[Denominator[n1 /. s3] == Denominator[n3 /. s3]]
```

Out[29]= True

In[30]:=
```
(*For two temperatures*)
```

In[31]:=
```
dif = Q - n1 R1 - n2 R2
mf2 = dif.dif
```

Out[31]= $\{q[1]-n1\,r1[1]-n2\,r2[1],\ q[2]-n1\,r1[2]-n2\,r2[2],\ q[3]-n1\,r1[3]-n2\,r2[3],\ q[4]-n1\,r1[4]-n2\,r2[4],\ q[5]-n1\,r1[5]-n2\,r2[5]\}$

Out[32]= $(q[1]-n1\,r1[1]-n2\,r2[1])^2+(q[2]-n1\,r1[2]-n2\,r2[2])^2+(q[3]-n1\,r1[3]-n2\,r2[3])^2+(q[4]-n1\,r1[4]-n2\,r2[4])^2+(q[5]-n1\,r1[5]-n2\,r2[5])^2$

In[33]:=
```
d1 = 1/2 Simplify[∂n1 mf2]
d2 = 1/2 Simplify[∂n2 mf2]
```

Out[33]= $-q[1]\times r1[1]-q[2]\times r1[2]-q[3]\times r1[3]-q[4]\times r1[4]-q[5]\times r1[5]+n1\left(r1[1]^2+r1[2]^2+r1[3]^2+r1[4]^2+r1[5]^2\right)+n2\,r1[1]\times r2[1]+n2\,r1[2]\times r2[2]+n2\,r1[3]\times r2[3]+n2\,r1[4]\times r2[4]+n2\,r1[5]\times r2[5]$

Out[34]= $-q[1]\times r2[1]+n2\,r2[1]^2-q[2]\times r2[2]+n2\,r2[2]^2-q[3]\times r2[3]+n2\,r2[3]^2-q[4]\times r2[4]+n2\,r2[4]^2-q[5]\times r2[5]+n2\,r2[5]^2+n1\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])$

In[35]:=
```
Collect[d1, {n1, n2}]
Print[]
Collect[d2, {n1, n2}]
```

Out[35]= $-q[1]\times r1[1]-q[2]\times r1[2]-q[3]\times r1[3]-q[4]\times r1[4]-q[5]\times r1[5]+n1\left(r1[1]^2+r1[2]^2+r1[3]^2+r1[4]^2+r1[5]^2\right)+n2\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])$

Out[37]= $-q[1]\times r2[1]-q[2]\times r2[2]-q[3]\times r2[3]-q[4]\times r2[4]-q[5]\times r2[5]+n1\,(r1[1]\times r2[1]+r1[2]\times r2[2]+r1[3]\times r2[3]+r1[4]\times r2[4]+r1[5]\times r2[5])+n2\left(r2[1]^2+r2[2]^2+r2[3]^2+r2[4]^2+r2[5]^2\right)$

In[38]:=
```
(*Check*)
Simplify[d1 == -Q.R1 + n1 R1.R1 + n2 R1.R2]
Simplify[d2 == -Q.R2 + n1 R2.R1 + n2 R2.R2 ]
```

Out[38]= True

Out[39]= True

In[40]:=
```
s2 = FullSimplify[Solve[{
      QR1 == n1 R1R1 + n2 R1R2 ,
      QR2 == n1 R1R2 + n2 R2R2 },
     {n1, n2}]][[1]]
```

Out[40]= $\left\{n1 \to \frac{QR2\ R1R2 - QR1\ R2R2}{R1R2^2 - R1R1\ R2R2},\ n2 \to \frac{QR2\ R1R1 - QR1\ R1R2}{-R1R2^2 + R1R1\ R2R2}\right\}$

In[41]:=
```
(*Check the solution with the substitututions*)
subst = {QR1 → Q.R1, QR2 → Q.R2, R1R1 → R1.R1, R1R2 → R1.R2, R2R2 → R2.R2};
Simplify[d1 /. s2 /. subst]
Simplify[d2 /. s2 /. subst]
```

Out[42]= 0

Out[43]= 0

In[44]:=
```
CForm[n1 /. s2]
```

Out[44]//CForm=
```
(QR2*R1R2 - QR1*R2R2)/(Power(R1R2,2) - R1R1*R2R2)
```

In[45]:=
```
CForm[n2 /. s2]
```

Out[45]//CForm=
```
(QR2*R1R1 - QR1*R1R2)/(-Power(R1R2,2) + R1R1*R2R2)
```

In[46]:=
```
(*Check that the denominators in the equations for n1, n2, n3 are equal*)
Denominator[n1 /. s2] == -Denominator[n2 /. s2]
```

Out[46]= True

In[47]:=
```
(*For one temperature*)
```

In[48]:=
```
dif = Q - n1 R1
mf1 = dif.dif
```

Out[48]= {q[1] - n1 r1[1], q[2] - n1 r1[2], q[3] - n1 r1[3], q[4] - n1 r1[4], q[5] - n1 r1[5]}

Out[49]= $(q[1] - n1\ r1[1])^2 + (q[2] - n1\ r1[2])^2 + (q[3] - n1\ r1[3])^2 + (q[4] - n1\ r1[4])^2 + (q[5] - n1\ r1[5])^2$

In[50]:=
```
d1 = 1/2 Simplify[∂n1 mf1]
```

Out[50]= $\frac{1}{2}\left(-2\ q[1] \times r1[1] - 2\ (q[2] \times r1[2] + q[3] \times r1[3] + q[4] \times r1[4] + q[5] \times r1[5]) + 2\ n1\ \left(r1[1]^2 + r1[2]^2 + r1[3]^2 + r1[4]^2 + r1[5]^2\right)\right)$

In[51]:=
```
Collect[d1, {n1, n2}]
```

Out[51]=
$$n1\left(r1[1]^2+r1[2]^2+r1[3]^2+r1[4]^2+r1[5]^2\right)+\frac{1}{2}\left(-2\,q[1]\times r1[1]-2\left(q[2]\times r1[2]+q[3]\times r1[3]+q[4]\times r1[4]+q[5]\times r1[5]\right)\right)$$

In[52]:=
```
Simplify[d1 == -Q.R1 + n1 R1.R1 ]
```

Out[52]=
```
True
```

In[53]:=
```
s1 = FullSimplify[Solve[{
      QR1 == n1 R1R1 },
    {n1}]][[1]]
```

Out[53]=
$$\left\{n1\to\frac{QR1}{R1R1}\right\}$$

In[54]:=
```
(*Check the solution with the substitututions*)
subst = {QR1 → Q.R1, R1R1 → R1.R1};
Simplify[d1 /. s1 /. subst]
```

Out[55]=
```
0
```

In[56]:=
```
CForm[n1 /. s1]
```

Out[56]//CForm=
```
QR1/R1R1
```