Basics of our Vulcan Magnetic Field Measurements

The megagauss magnetic fields generated in the intense laser-plasma interaction induce a birefringence in the plasma (discussed in detail later), resulting in a change in the state of polarization of the initially linearly polarized incident probe pulse. For a normally incident probe, the change in the state of polarization is manifested as a Faraday rotation ψ due to the axial component of the magnetic field as well as the introduction of an ellipticity χ in accordance with the Cotton-Mouton effect due to the azimuthal component of the magnetic field, as shown in Figure 1.

In general, a complete description of the state of polarization of an electromagnetic wave is given by its Stokes' vector [1], defined as

$$\mathbf{s} \equiv \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \equiv I_0 \begin{pmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{pmatrix}.$$

It can be shown [2, 3] that the evolution equation (along the z direction) for the Stokes' vector in the magnetized plasma is given as[1]

$$\frac{d\mathbf{s}(z)}{dz} = \mathbf{\Omega}(z) \times \mathbf{s}(z),$$

where

$$|\mathbf{\Omega}| \equiv -\frac{\omega}{c}(\mu_o - \mu_x),$$

ω being the frequency of the incident laser pulse and μ_o and μ_x the refractive indices of the ordinary O- and the extraordinary X-waves respectively.

The refractive index of the ordinary O-wave is given by the usual relation

$$\mu_o \equiv \sqrt{1 - \frac{\omega_p^2}{\omega^2}},$$

[1] Essentially, the state of polarization may also be represented by the Poincare sphere [1, 3], which is a sphere of unit radius in the (s_1, s_2, s_3) space, where a given state of polarization is uniquely represented by a point on the Poincare sphere, with latitude 2χ and longitude 2ψ. The evolution of the Stokes' vector is then represented on the Poincare sphere by a rotation about an axis passing through the points representing the characteristic orthogonal polarization vectors and is thus described by the aforesaid evolution equation.

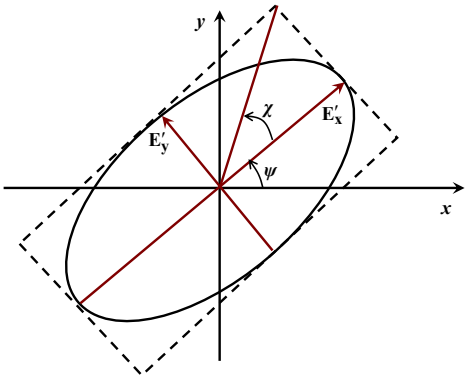


Figure 1: The polarization ellipse

where ω_p is the plasma frequency. On the other hand, the refractive index μ_x of the extraordinary X-wave however depends on the ambient megagauss magnetic field B via the cyclotron frequency $\omega_c = eB/m$, e and m being the charge and the mass of an electron, according to the relation

$$\mu_x \equiv \sqrt{1 - \frac{\omega_p^2}{\omega^2}\left(\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2 - \omega_c^2}\right)}.$$

The different refractive indices lead to the accumulation of different phases by the two characteristic waves as the probe traverses across the plasma, which results in the induced ellipticity. Essentially, the cut-off for the O-wave is at $\omega = \omega_p$, and hence it reflects from the usual critical density surface. However, the externally incident X-wave is reflected from the so-called right-hand cutoff [4] at

$$\omega_R = \frac{1}{2}\left(\omega_c + \sqrt{\omega_c^2 + 4\omega_p^2}\right).$$

Thus, for a given incident laser frequency ω, the X-wave reflects from a density lower than the critical surface density, from which the O-wave reflects, the difference between the two depending on the magnetic field B. Obviously, the density profile decides the difference in the turning points and therefore, what is required for the estimation of the magnetic field from the measured ellipticity is the scale length of the plasma (at the rearside, for our measurements). Essentially, the plasma density profile is modelled by assuming that the plasma expands into vacuum at the ion sound speed c_s in a self-similar fashion [5]. This yields the exponential density profile, where the scale length is given as $L = c_s t$, where t is the time and the expansion velocity c_s is given by the expression

$$c_s \equiv \sqrt{\frac{Zk_BT}{m_i}},$$

T being the bulk plasma temperature, Z the degree of ionization and m_i the mass of the ion. For example, Gremillet *et al.* [6] and Malka *et al.* [7] specify the bulk plasma temperature of common targets (such as 100 μm thick fused silica and 50 μm thick aluminium foil) as typically of the order of a few eVs at the target rear at laser intensities of $10^{18} - 10^{19}$ W/cm^2.

In summary, the measurement of the Stokes' parameters of the reflected probe, on comparison with the incident probe, yields the ellipticity and the Faraday rotation (negligibly small in our case) induced in the probe due to the presence of the magnetic fields in the region of the plasma through which the probe has traversed. The code which deduces the magnetic field from the ellipticity essentially solves the above evolution equation numerically and iteratively to evaluate the magnetic field required to generate the ellipticity observed in the experiment. The entire plasma box is divided into several cells, where the output Stokes' vector of one cell is fed as the input Stokes' vector of the next cell and hence the evolution of the Stokes' vector is monitored over the entire length of the plasma traversed by the probe.

References

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